The relationship between the Cole-Cole exponent and the mean relaxation time

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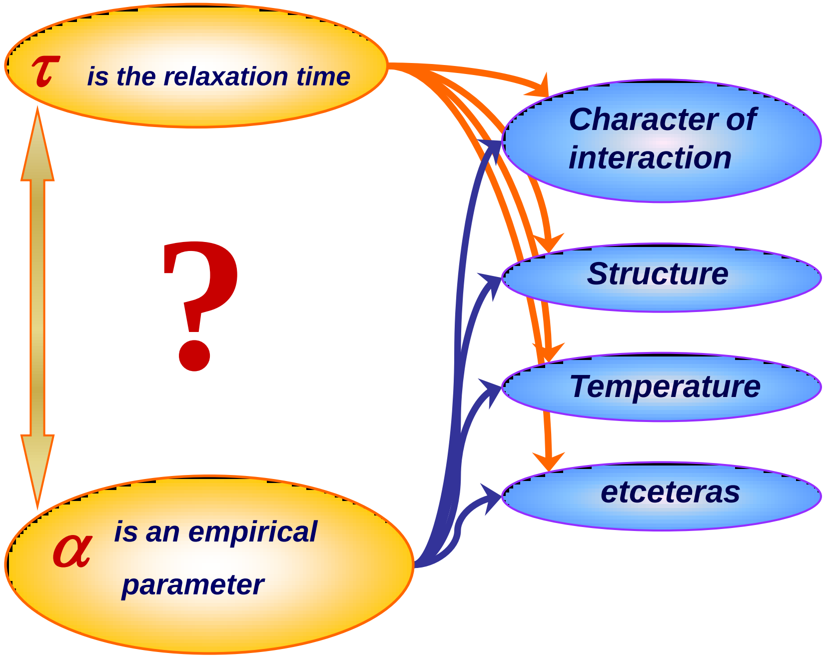
Non-exponential relaxation

empirical

Cole-Cole law

$$\varepsilon = \varepsilon_\infty + \frac{\Delta\varepsilon}{1+(i\omega\tau)^{\alpha}}$$

1941 year



τ **is the relaxation time**

?

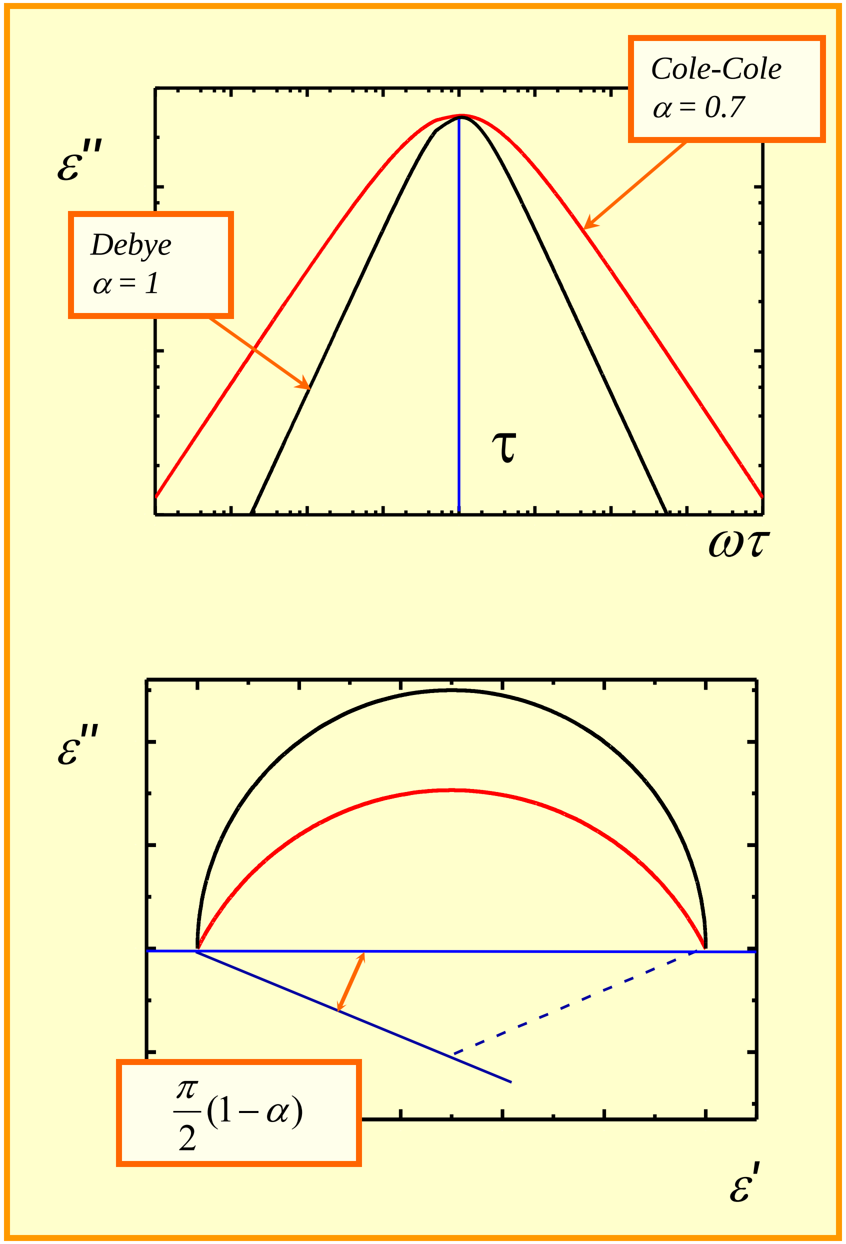
α **is an empirical parameter**

Character of interaction

Structure

Temperature

etceteras



The Cole-Cole memory function

$$\frac{d}{dt}f(t) = -\int_0^t M(t-t')f(t')\,dt'$$

$$\tilde{M}(i\omega) = (i\omega)^{1-\alpha}\tau^{-\alpha}$$

$$\boxed{\frac{df}{dt} = -\tau^{-\alpha}D_0^{1-\alpha}[f(t)]}$$

Averaging over a cooperative domain

$$\boxed{\alpha = d_f}$$

$$m_\delta(t) = \sum\delta(t-t_i) \quad \Longrightarrow \quad \langle M(t) \rangle$$

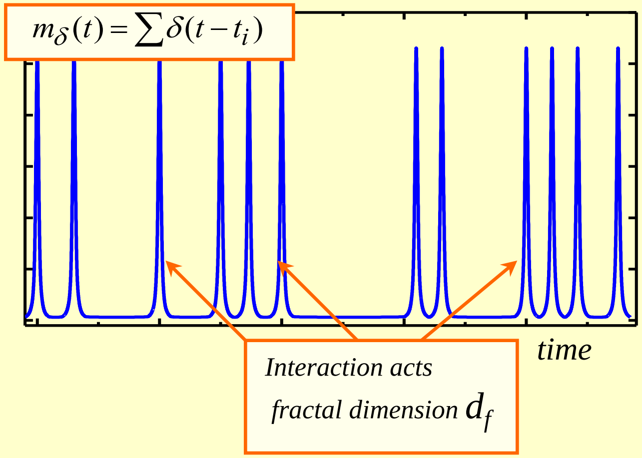
Microscopic scale

Macroscopic scale

R. R. Nigmatullin, Ya. E. Ryabov, *Physics of the Solid State*, **39** (*1997*)

A. Le Mehaute, R. Nigmatullin, L. Nivanen, *Fleches du Temps et Geometry Fractale* (Hermez, Paris, *1998*)

Microscopic memory function of a single relaxator

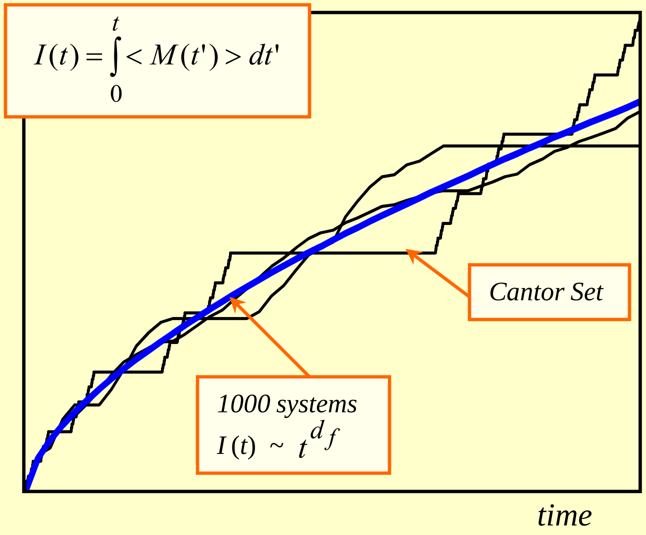


$m_\delta(t) = \sum\delta(t-t_i)$

Interaction acts

fractal dimension d_f

Macroscopic memory function of a cooperative domain



$I(t) = \int_0^t \langle M(t') \rangle\, dt'$

Cantor Set

1000 systems

$I(t) \sim t^{d_f}$

Scaling relationships

$$\boxed{\alpha = d_f = \frac{\ln(N)}{\ln(\xi)}}$$

$$\xi = \frac{\tau}{\tau_0}$$

ξ *is the dimensionless time interval size and N is the number of relaxation acts in that interval*

$$\Downarrow$$

$$N = G\left(\frac{R}{R_0}\right)^{d_G}$$

N *is of relaxation acts on microscopic level for a cooperative domain,*

or by the ergodic assumption is the number of points where the relaxing units are interacting with the statistical reservoir

$$\boxed{\alpha = \frac{d_G}{2}\frac{\ln(\tau\,\omega_s)}{\ln(\tau/\tau_0)}}$$

$$R^2 = 2d_E D_s \tau$$

Einstein-Smoluchowski theory

τ_0 *is the cutoff time of the scaling in time domain*

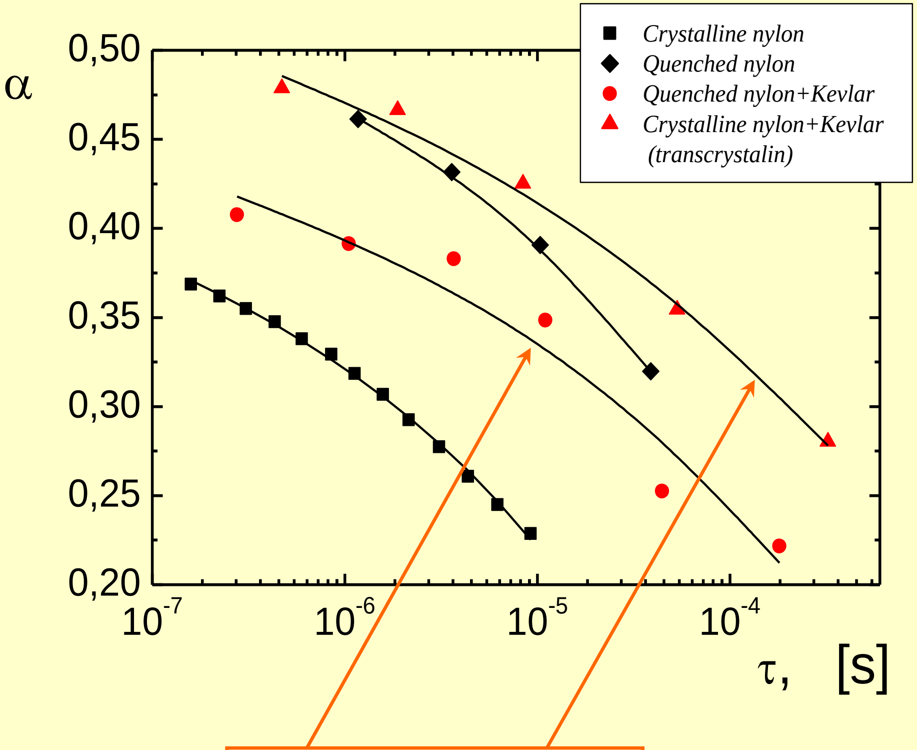
d_G *is a spatial fractal dimension of the points set where relaxing units are interacting with the statistical reservoir*

$$\omega_s = \frac{2d_E D_s}{R_0^2}\sqrt{G^{d_G}}$$

is the characteristic frequency of the self-diffusion process

Composite material T is not constant

Sample	d_G	$\tau_0 \times 10^3$ [s]	$\omega_s \times 10^{-3}$ [Hz]
QN	1.4	7.1	2.2
CN	1.2	8.7	8.9
QNM	1.3	2.4	5.2
CNM	1.5	1.6	1.4



Samples with Kevlar fibers

$$\boxed{\alpha = \frac{d_G}{2}\,\frac{\ln(\tau\,\omega_s)}{\ln(\tau/\tau_0)}}$$

H. Nuriel, N. Kozlovich, Y. Feldman, G. Marom *Composites: Part A* ***31*** (2000) *p. 69*

Water-polymer mixtures *T=25 ^{0}C (constant)*

Sample	d_G	$\tau_0 \times 10^{12}$ [s]	$\omega_s \times 10^{-11}$ [Hz]	$D_s \times 10^9$ [m^{2}s^{-1}]
PVA	1.56	7.18	1.47	3.31
PAIA	1.43	6.46	1.74	3.92
PAA	1.12	6.34	2.08	4.68
PEI	1.33	4.89	2.67	6.01
PEG	1.54	4.45	2.78	6.26
PVME	1.38	3.58	4.24	9.54
PVP	0.92	0.72	241	542

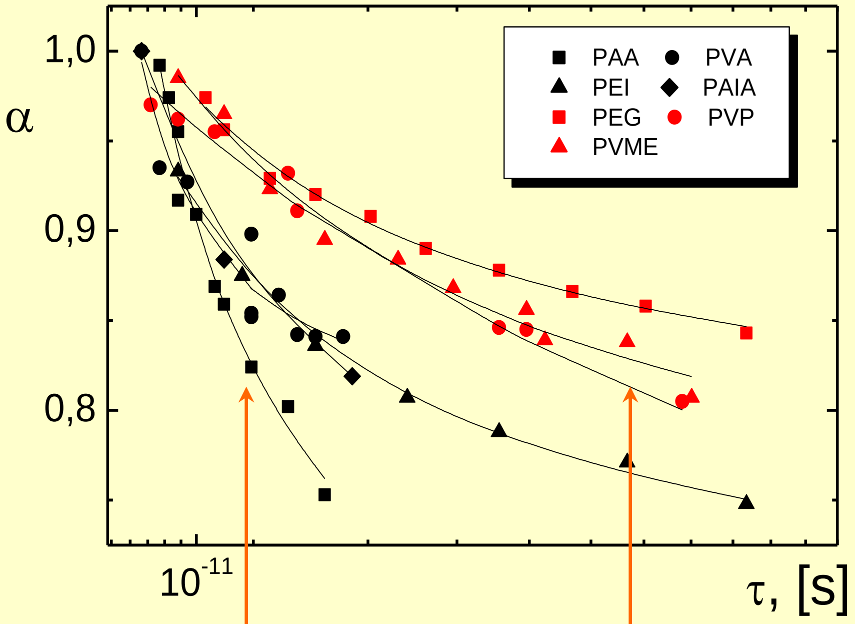
$$\boxed{\alpha = \frac{d_G}{2}\frac{\ln(\tau\,\omega_s)}{\ln(\tau/\tau_0)}}$$

bulk water

Microscopic relaxation time $\tau_0 \approx 0.8 \times 10^{12}\ s$

Macroscopic relaxation time $\tau \approx 8.3 \times 10^{12}\ s$

Self diffusion coefficient $D_s \approx 2.6 \times 10^{-9}\ m^2s^{-1}$



Hydrophilic mixtures

Hydrophobic mixtures

N. Shinyashiki, S. Yagihara, I. Arita, S. Mashimo,
Journal of Physical Chemistry, B ***102*** *(1998) p. 3249*

Conclusions

I *The Cole-Cole scaling parameter depends on the features of interaction between the system and the thermostat.*

II *The Cole-Cole scaling parameter and the relaxation time are directly connected to each other.*

III *From the dependence of the α parameter on the relaxation time, the structural parameters can be defined.*